A Least Squares Solution for Use in the Six-Port Measurement Technique

by

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Abstract

Because it permits the use of simple amplitude rather than complex ratio detectors, the six-port technique has captured the attention of the microwave community. Here, the determination of complex reflection coefficient, (Γ) is obtained from the intersection of three circles, in the complex plane, whose radii are obtained from the amplitude detectors. As a practical matter, however, these circles will not intersect in a point because of measurement error.

This paper addresses two questions: 1) How does one choose Γ in this context, and 2) What can be inferred about the system accuracy from the extent of this intersection failure?

Summary

I. INTRODUCTION

The so called "six-port technique" provides for the measurement of both phase and amplitude while using detectors which respond only to magnitude. [1] Moreover, the accuracy realized in a prototype network analyzer [2] based upon this technique, is virtually without precedent. [3] For these reasons, this technique has captured the attention of the microwave community.

As explained in [1], it is convenient to visualize the six-port operation in terms of a diagram in the complex plane where the value of an unknown reflection coefficient, Γ_ℓ, is obtained from the intersection of three circles, whose radii are determined by the response of the amplitude detectors, while the positions of the centers are determined primarily by the parameters of the six-port network and its detectors, and to a small extent by the detector readings.

In practice, because of measurement error, the three circles will not intersect in a point, and some method must be devised for choosing the most probable value of Γ_ℓ in this context. Moreover, when properly interpreted, the extent of the intersection failure becomes a useful monitor of the system performance.

II. FORMULATION

While it is possible to write an expression for Γ_ℓ as a function of the responses of the six-port sidearm detectors, P_3, P_4, P_5, P_6, it has proven convenient [4] to introduce the parameter w whose value is given by the solution of the equations,

$$|w|^2 = P_3/P_4 \qquad (1)$$

$$|w-w_1|^2 = \zeta P_5/P_4 \qquad (2)$$

$$|w-w_2|^2 = \rho P_6/P_4 \qquad (3)$$

where w_1, w_2, ζ, ρ are parameters (whose values are assumed to be known) of the six-port and its detectors. Following this, Γ_ℓ is given by

$$\Gamma_\ell = \frac{\alpha w+\beta}{\gamma w+1} \qquad (4)$$

where α, β, γ are additional six-port parameters.

Returning to (1), (2), (3) it will be recognized that w is given by the intersection of three circles, with centers at 0, w_1, w_2, and of radii $\sqrt{P_3/P_4}$, $\sqrt{\zeta P_5/P_4}$, $\sqrt{\rho P_6/P_4}$ respectively. Thus, in practice, it is w, rather than Γ_ℓ, which is obtained from the intersection of three circles, and the practical problem is that of choosing w when the circles fail to intersect.

Let k represent the true value of P_4. Then the fractional error ε_4 in the measured value, P_4, is given by[1]

$$\varepsilon_4 = \frac{P_4-k}{P_4} \qquad (5)$$

The corresponding errors ε_3, ε_5, ε_6, in P_3, P_5, P_6 are given by

$$\varepsilon_3 = \frac{P_3-k|w|^2}{P_3} \qquad (6)$$

$$\varepsilon_5 = \frac{P_5-k|w-w_1|^2/\zeta}{P_5} \qquad (7)$$

$$\varepsilon_6 = \frac{P_6-k|w-w_2|^2/\rho}{P_6} \qquad (8)$$

Now, if both k and w were known, then E, where

$$E = \tfrac{1}{4}\sum_{i=3}^{6} \varepsilon_i^2 \qquad (9)$$

represents the mean square error in the detector readings for a given measurement. In actual practice, this suggests choosing k and w in such a way that E is a minimum.

[1]Strictly speaking, the denominator should be "k" rather than "P_4". However, the solution to the problerm as thus formulated is much simpler and the difference in end result negligible.

Following this, E is an indication of the overall detector error.

The minimization of E, as a function of k and w is a fairly straightforward exercise in the differential calculus which leads to a pair of simultaneous cubic equations. These may be easily solved by numerical methods.

III. CONCLUSION

The foregoing technique, which has been implemented for use in the NBS six-port systems, provides for the optimal use of the redundant information, and a continuous monitor on the performance of the power meters which make the amplitude measurements.

REFERENCES

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